Dispersion, Coverage, and the Rational Investor: Using Analysts' Earnings Forecasts to Predict Stock Returns

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Abstract

Researchers have often used intrinsic properties of stocks such as earnings per share ratios and dividend yields to predict stocks' future performances. In their paper *Differences of Opinion*, Diether et al. (2002) propose a new, non-intrinsic predictive variable based on analyst earnings forecasts. Using their methods, we validate the claim that greater differences of opinion in analyst predictions of future stock returns tracks poorer future returns of a stock and that small stocks and poorly performing stocks are especially affected. Going further, we also found that the dispersion effect was greatest for mid-level analyst coverage and that market performance was negatively related to the dispersion effect. Also, by considering more realistic and rational investor behavior, we are able to more confidently reject the hypothesis that dispersion effects can be explained by Fama-French factors. However, inclusion of more realistic rational investor behavior led to different portfolio behavior, and decreased explanatory power of dispersion, suggesting that the dispersion effect is not as robust as previously thought.

Introduction

Over the years, there have been many attempts to predict the future performance of stocks. Early researchers tested the use of the historical performance of stocks as indicators for future returns. Currently, stocks' intrinsic properties, such as interest rates, dividend yields (dividends over stock price), book-to-market ratios (the book value of a stock over the market value of the stock), and earnings-price ratios have begun to be examined as predictive factors. Karl B. Diether et al. (2002) extend the list of predictive variables beyond a stock's intrinsic properties by introducing a new variable called dispersion, measuring the difference in opinion regarding the future returns of that stock. It is assumed that the difference in opinion on the earnings forecasts made by analysts reflect the difference in opinion on the earnings forecasts made by investors overall.

The logic behind the use of dispersion as a predictive variable by Karl B. Diether et al. (2002) is as follows: the best indicator of a stock's price is the average opinion (that is, average prediction of future returns) of all investors since this ultimately determines the *market* value of the stock. If there is high dispersion, than there are many investors who believe the stock have high value, and also many who believe it has low value. Those who believe it has a high value, the optimists, invest in the stock regardless of price because they believe the price will continue to grow. On the other end of the spectrum, because so many people believe the stock will do poorly, there will be an expected increase in demand for short-sales; thus driving short-sale costs higher. Therefore, due to the higher short-sale costs we expect pessimists to be less likely to short the stock. Since pessimists are kept out of the market by these high short-sale costs, the price of the stock is artificially inflated relative to the average opinion of all investors. We expect that the higher the dispersion, the more artificially high the price will be and the worse actual future performance will be. Thus,

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those stocks with high dispersion in forecasted returns are expected to have lower future returns than those stocks with low dispersion. The incentive system for analysts, which discourages analysts who predict poor performance from publishing, is analogous to the high short-sale costs that exclude pessimistic investors who would otherwise lower the valuation of a stock.

We thus conclude that our original assumption that analyst predictions are a proxy for investor opinions is a reasonable one, and our main goal of this paper is to test Karl B. Diether et al's (2002) findings that dispersion is a good predictive variable of a stock's future performance in many scenarios. In this paper we first outline the hypotheses that we test, next perform Fama-French multifactor time-series tests to determine the explanatory power of dispersion on returns, and finally present portfolio strategies to examine the relationship between dispersion and returns. This work also extends the work of Karl B. Diether et al. (2002) by considering whether the magnitude of the relationship between dispersion and stock returns is affected by overall performance of the market and the number of analyst estimates. In addition, given the sophistication of today's investors it is possibly too simplistic to have investors always longing stocks and hoping for higher future returns as Karl B. Diether et al. (2002) do in their regression and most of their portfolio results. Instead, a more realistic investor, which we term as the *rational* investor, would also short stocks if the stock was expected to fall in value. Since we use analyst opinions as a proxy for investor opinions in general, we can model the average behavior of a rational investor by using the average analyst estimate as an indicator of the average rational investor's expected future valuation of a stock. For example, if the mean analyst estimate for a stock had a negative earnings-per-share (EPS) value we would expect the average rational investor to short that specific stock. This paper examines how such a rational investor's behavior alters Karl B. Diether et al. (2002) findings.

Methodology

Hypotheses

In this paper, we use empirical data to analyze the predictive ability, if any, of dispersion in analysts' earnings forecasts on the cross section of future stock returns. There are three possible hypotheses: First, dispersion in forecasts is a proxy for differences of opinion among investors. The logic behind this hypothesis was summarized in the introduction. To reiterate, when there is a large difference of opinion in a stock's valuation, equity prices tend towards the more optimistic valuations, leading to lower future returns as prices converge to a stock's true valuation. The larger the difference in valuations of a stock, the lower the future returns of a stock. The second hypothesis is that dispersion in forecasts is indeed a proxy for differences of opinion among investors, but that market prices will be unaffected by greater differences of opinion among investors. This hypothesis was proposed under the assumption that arbitrageurs behave according to certain principles. They are perfectly rational and don't face any high short-sale costs, so any pessimistic investors would voice their opinion on a stock by shorting it. The arbitrageurs are also risk-neutral, so they care only about the expected value of a stock's return, and not the risk involved, and are motivated to maximize profits because they are competitive. Clearly, stocks will then be sold at their actual expected values and will be bought at that same value, regardless of dispersion. Thus, this model obviously predicts no relationship between the dispersion in analysts' earnings forecasts and future returns. The last hypothesis is that dispersion in forecasts is a proxy for risk. Investors view a stock with greater dispersion as a riskier investment and will demand greater compensation for this risk. Greater dispersion in analysts' forecasts makes future returns more difficult to predict, so stocks with higher dispersion should naturally earn higher future returns. This model predicts that increasing dispersion should lead to increasing future returns. To summarize, the first hypothesis posits a negative relationship between dispersion and future returns, the second, no relation, and the third, a positive relation.

Data Collection

We obtain our data from the Wharton Research Data Services database that holds subscriptions to the Center for Research in Security Prices (CRSP), COMPUSTAT, and the Institutional Brokers Estimate System (I/B/E/S).

We obtain the analysts' earnings estimates from I/B/E/S from January 1983 to November 2000 as Karl B. Diether et al. (2002) did. However, in the adjusted dataset as stocks undergo splits, I/B/E/S rounds analyst earnings per share estimates to the nearest cent. For example, if a stock were to split ten-fold, I/B/E/S would record both 10 cent and 14 cent estimates (the example given in the Karl B. Diether et al. (2002)) as 1 cent estimates, indicating a standard deviation of zero in the estimates, when clearly this is not so. Thus it is important that we use returns estimates based on the number of shares at the time the estimate was made, not the current number of shares so we use the unadjusted data set. The adjusted data would have led to data bias towards unusually high returns for stocks with low dispersion in estimates, since we would observe more firms that had done well (that is, well enough to split) after the estimates were made. Therefore we choose to use unadjusted I/B/E/S data. Dispersion was calculated as the standard deviation in analysts' earnings forecasts over the absolute value of the mean of those forecasts. Those securities with a mean forecast of zero in time period t are automatically regarded as being in the highest level of dispersion.

We obtain returns for I/B/E/S stocks in a given month from the CRSP Monthly Stocks Combined File, which covers the New York Stock Exchange (NYSE), the American Exchange (AMEX), and NASDAQ. For each stock in the CRSP data set, we match monthly I/B/E/S summary statistics with the monthly returns for that stock. COMPUSTAT data on those stocks' book-to-market ratio was then obtained. We only considered stocks that were covered by two or more analysts in order to be able to calculate the standard deviation in estimates needed for calculating dispersion. Also, stocks with prices below five dollars were excluded from the final dataset in order to prevent biases being introduced from bid-ask bounce phenomenon and the presence of smaller illiquid stocks [9]. However, not all stocks covered in CRSP and COMPUSTAT are covered by I/B/E/S because analysts typically only follow a fraction of the total number of stocks in the market, and more frequently track larger and more established companies. Thus when the cross section of the CRSP, COMPUSTAT, and IBES is taken, a large amount of data is inevitably lost. As Karl B. Diether et al. (2002) note, this raises the concern that larger and more stable companies are overrepresented. We considered creating counterfactual data to represent smaller and more unstable stocks in the same proportion that they were excluded from the merged I/B/E/S, COMPUSTAT and CRSP data set, but this would have led to too much error in our results because historically, smaller and more unstable stocks exhibit much greater deviation in expected returns.

Our data set produces results that closely track the results in Karl B. Diether et al. (2002) in most instances. However, they are not identical, most likely due to slight differences in data collection. For example, in the original paper the authors use market capitalization data from the third Thursday of each month to form some tables, while using data from the last day of each month in others. For consistency in methodology, we only collect data from the last day of the month. However, because we eventually divide all stocks into quintiles based on market capitalization, this discrepancy is does not affect our results since a stock's returns will usually not change drastically enough to jump into a different market capitalization quintile within one or two weeks. Our most likely source of error comes from the merging of CRSP, I/B/E/S, and COMPUSTAT data. Because the data sets use differing stock identifiers, we created several of our own that gave each stock-month a unique identifier. However, there are many ways to create identifiers to merge datasets, and given the large data loss after merging, the method of identification plays a large role in the resulting data set. Unfortunately, we were unable to learn from Karl B. Diether et al. (2002) their exact method of merging datasets.

Regression Tests

In this paper, we first analyze our results using Fama-French multifactor time-series regression tests. Our regression tests will focus on whether the Fama-French models, the standard models in finance for predicting returns, can explain away the effect of dispersion on other factors. We provide an overview of the underlying logic behind the precursor to the Fama-French model, the capital asset pricing model (CAPM), as well as the Fama-French models in the Appendix. After briefly introducing the Fama-French three and four factor models we present our replication results of Karl B. Diether et al. (2002) regression tests and then extend the regression results using the rational investor theory outlined previously.

The Three Factor Model The Fama-French[6] multifactor asset pricing model is based on the simple assumption that an individual security's return is closely related to its risk. In the more simple capital asset pricing model (CAPM) discussed in the *Appendix*, we see that expected earnings on a security less a risk-free rate of interest is equal to some risk factor times the expected earnings of the market itself less the same risk-free rate. Fama and French (1996) expanded CAPM to include more factors beyond market returns $(R_M - R_F)$, namely a security's size in relation to other securities' (SMB) and the security's book-to-market ratio in relation to other securities' (HML) [6]:

$$E(R_{it} - R_{Ft}) = b_{iM}(R_M - R_F) + s_i SML_t + h_i HML_t + \epsilon_t$$

where the left hand side of the equation is the expected return of security i in period t less the risk-free rate of return in the same period and the left hand side are the factors: market premium, size premium, and value premium. The coefficients are the relative sensitivities of a stock's returns to the different factors. Fama and French's data suggests that a security's returns are more completely explained by three factors, rather than just one. Again, just as in the CAPM, the stochastic component is given by:

$$\epsilon_t \sim \text{Normal}(\mu_t, \sigma_t)$$

and the systematic component is given by

$$\mu_t = b_{iM}(R_M - R_F) + s_i SML_t + h_i HML_t$$

and σ_t is the standard deviation in overall market returns in the time period t (See the *Appendix* for a more in depth discussion of the systematic and stochastic components of CAPM).

In the multifactor time-series tests, we find the size and value premiums for each month in addition to the market premium represented in the CAPM and using return data for stocks from February 1983 to December 2000. We then calculate the dispersion of each stock for each month and then assign the stock into a quintile based on its level of dispersion. Then we average the returns time series for each date and dispersion quintile and perform five separate regressions over the five dispersion based portfolios to find the relative sensitivities (coefficients) of a stock's returns to the three factors. Next, we perform the same regression, this time adding a fourth explanatory variable called the momentum premium (UMD). This premium uses the fact that stocks that have performed well in the past typically continue to do so, while stocks that have performed poorly in the past also typically continue to do so in the future [8]. We construct the momentum premium each month by taking the difference between the compounded returns on a portfolio with stocks with high returns from month $t - 12$ to $t - 2$ and stocks with low returns in that same period, yielding a four factor model:

$$E(R_{it} - R_{Ft}) = b_{iM}(R_M - R_F) + s_i SML_t + h_i HML_t + m_i UMD_t$$

and the stochastic component is given by:

$$\epsilon_t \sim \text{Normal}(\mu_t, \sigma_t)$$

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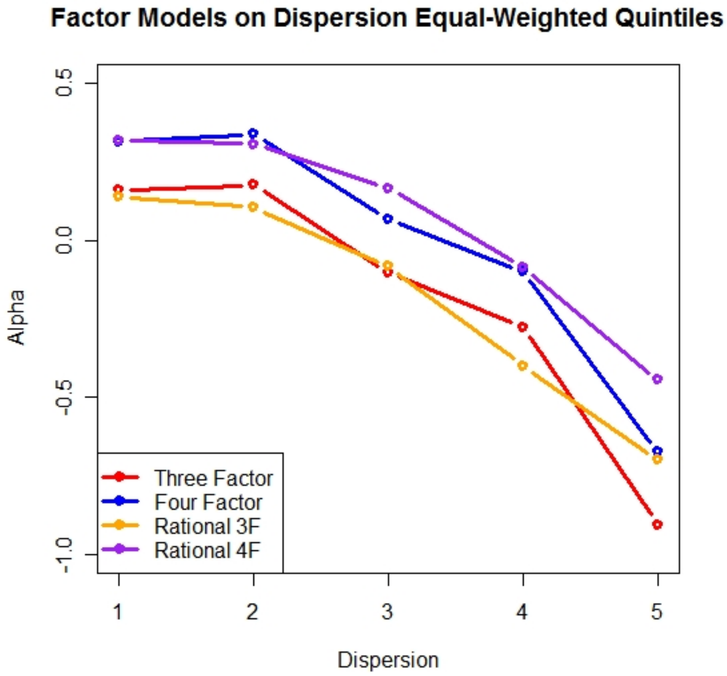
$$\mu_t = b_{iM}(R_M - R_F) + s_i SML_t + h_i HML_t + m_i UMD_t$$

and σ_t is the standard deviation in overall market returns in the time period t.

For both the three- and four-factor Fama-French time-series regressions, we are mainly concerned with the intercepts, or α values which is comprised of the systemic effect unexplained by the model and a stochastic error component. By creating portfolios based on dispersion quintiles, these α values estimate how much of a portfolio's returns cannot be explained by the regression factors. Thus large α values generally suggest greater uncertainty that the model's factors can adequately explain a portfolio's performance. The t-statistics are Newey-adjusted to account for autocorrelation [11]. As in Karl B. Diether et al. (2002), we find that α values decrease in both models as dispersion increases, becoming quite negative in the higher dispersion quintiles. To test whether the Fama-French model factors can explain away the dispersion effect of the portfolios we perform a GRS statistical test [7]. Karl B. Diether et al's (2002) GRS statistics of 5.51 and 2.62 for the three and four-factor models respectively allow them to reject the hypothesismodels' factors as being able to explain away the dispersion effect. While we do find the same trend of a decreasing GRS statistic as we move from the Fama-French three to four factor model, we find GRS statistics of 7.56 and 4.58 for the three and four-factor model respectively. These larger GRS statistics allow us to even more confidently reject the hypothesis that the three and four-factor models explain away the dispersion effect. The differences in the GRS statistics and large variations in coefficient values are most likely due to differences in data-parsing as discussed in the beginning of the Methodology section as our regression and statistical procedures are identical to Karl B. Diether et al (2002). Looking at the α values, which represent both the error and unexplained effects in the Fama-French multifactor model, while considering rational investment behavior, we find weaker t-statistics and smaller intercepts. This suggests that the α values are explained more by stochastic error and less by the Fama-French factors or dispersion under original investor behavior. However, stronger GRS statistics imply that the systemic unexplained portion of future stock returns is much more likely, so we conclude that dispersion explains away much less of future stock returns than we originally thought.

Now we perform the same Fama-French regression tests as above but with a rational investor behavior. Recall from the Introduction that we model the behavior of the average rational investor as that of someone who shorts stocks if the average analyst EPS estimates are negative. Using this rule we create a dataset with modified returns to adjust for shorted stocks, and perform regressions to test whether the Fama-French three and four-factor models can explain the away the effects of dispersion on an investor who shorts stocks. Here, we show a graph of the α values for both the three and four-factor models over the five dispersion quintiles as Karl B. Diether et al (2002) did as well as the α values from the rational consumer model:



[1]

α values do vary with a rational investor. However, the same α value trends discussed above are still present. What are of note are the higher GRS-statistics of 7.59 and 5.419 for the three and four-factor models respectively. While the increase in the GRS statistic value between a rational investor and the assumed investor behavior of Karl B. Diether et al. (2002) for the three-factor model is negligible, there is a significant increase (from 4.58 to 5.419) in the GRS statistic's rejection power of the four-factor model when rational investor behavior is factored in.

Portfolio Strategies

Portfolios Explained

The standard procedure in asset pricing research pioneered by Jegadeesh and Titman (2001) is to create portfolios of stocks with certain characteristics. For example, a portfolio might consist of all stocks with market capitalization in the top decile in a particular month, or five portfolios could be created by dividing all stocks in a given month into dispersion quintiles. The motivation behind the creation of portfolios is to draw conclusions about the average returns for stocks with certain characteristics and to reduce variability in returns. By creating portfolios of a variety of stocks and examining whole portfolio returns, noise from individual stocks has less effect and we can more reliably focus on the expected returns of one class of stocks.

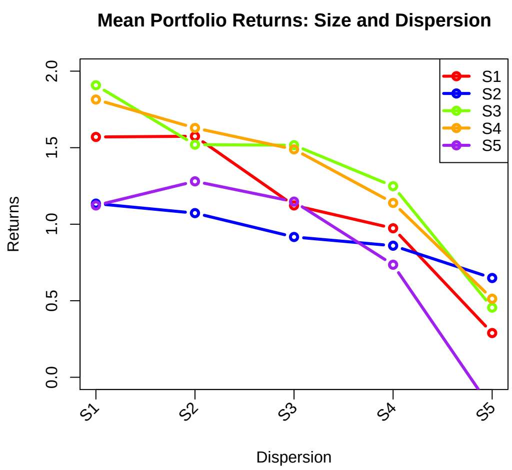
With portfolios with returns based on the Karl B. Diether et al's (2002) investor behavior we can then investigate overall trends in the relationship between predictive variables and average returns. More importantly, we can divide stocks into portfolios based on multiple characteristics, such as size *and* dispersion to investigate whether the trends in average returns are actually predicted by dispersion alone, and not simply the effect of size. We choose to first examine stock size, book-to-market ratio, and momentum as in Karl B. Diether et al. (2002). However, we also examine how sorting by two real-world variables that should impact investor behavior, the number of analyst estimates (coverage) and the market premium value (defined earlier

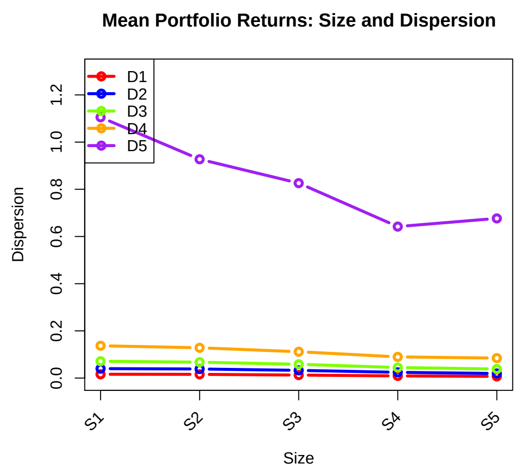
[1] See Tables I and II for coefficients

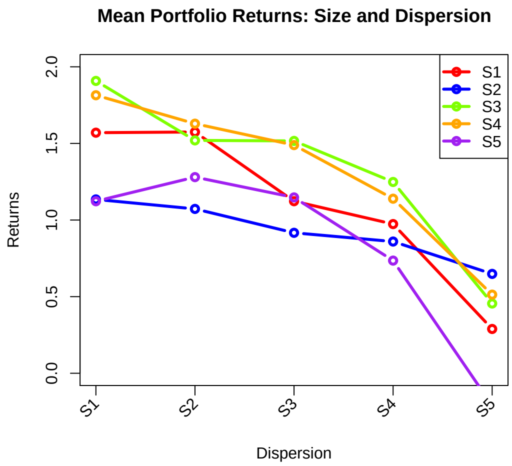
in the Fama-French model and termed here as market performance), effect dispersion's predictive ability on stock returns. In addition for all portfolio tests, we repeat the portfolio analyses with our previously defined rational investor behavior.

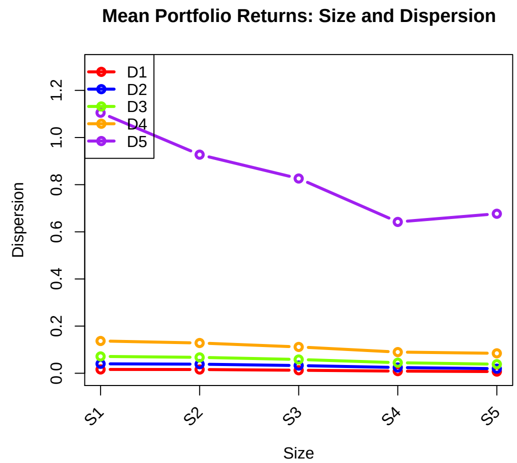
Size and Dispersion

We create our first portfolios based on a stock's market capitalization and dispersion. Again, dispersion is defined as the standard deviation of earnings forecasts divided by the absolute value of the mean earnings forecast. If the mean forecast is zero, then the stock's dispersion is automatically regarded as highest. Each month, we divide our set of stocks into five quintiles based on the dispersion in forecasts as of the month prior so that we investigate the relationship between the predictability of dispersion on return. The monthly returns are calculated as the equal-weighted average of the returns of the entire portfolio. We then create five sub-portfolios based on market capitalization, or size quintiles within the dispersion quintiles using the market capitalization of all stocks at the end of each month. As we can see in the graph, there is a strong negative correlation between dispersion and returns. Interestingly, the average returns of small stocks change particularly drastically as dispersion increases, but the negative correlation applies to all size quintiles. Compared to Karl B. Diether et al. (2002) the portfolio sorted by stock size and dispersion quintiles returned very similar trends as a whole. While returns do not exactly match this is expected since we know from our previous results that differing data parsing methods likely led to us developing a dataset with a different make-up of stocks. Notably, the one significant deviation is in the second quintile of size. Here returns deviate significantly from the trend that we would have expected from Karl B. Diether et al. (2002). Here are four graphs, the top two showing original investor behavior, and the bottom two showing rational investor behavior:









As we can see, both graphs are extremely similar, both exhibiting the same negative correlation between dispersion and returns across all size quintiles.

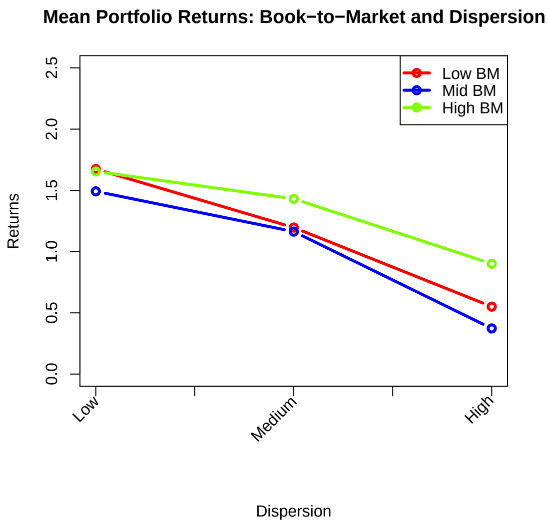
Size, Book-to-Market Ratio, and Dispersion

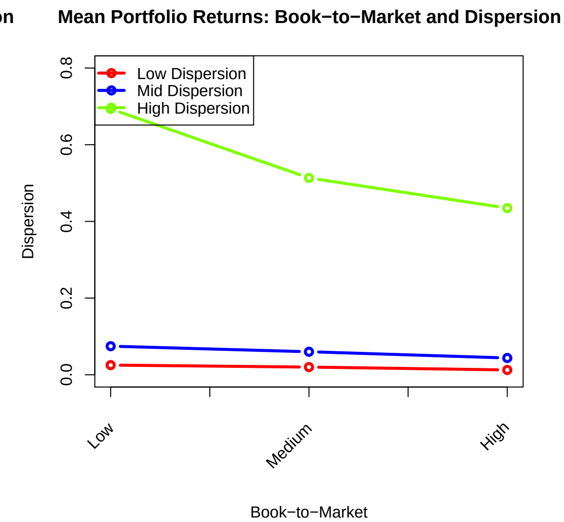
We create our next set of portfolios to investigate whether returns can be explained by another predictive variable, book-to-market ratio. Because stocks with low book-to-market ratios tend to have greater size, or higher levels of market capitalization, we also sort by size. First stocks are sorted by the level of market capitalization as reported in the previous month. Then three sub-portfolios are created within each size portfolio based on book-to-market ratio, and these sub-portfolios are further divided into three portfolios based on dispersion, resulting in 27 portfolios.

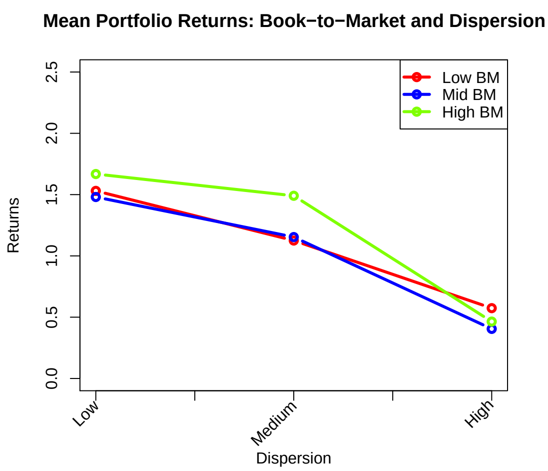
Book-to-market ratio, or the ratio of the book equity of a stock to the market equity, is defined in Diether et al. (2002) as:

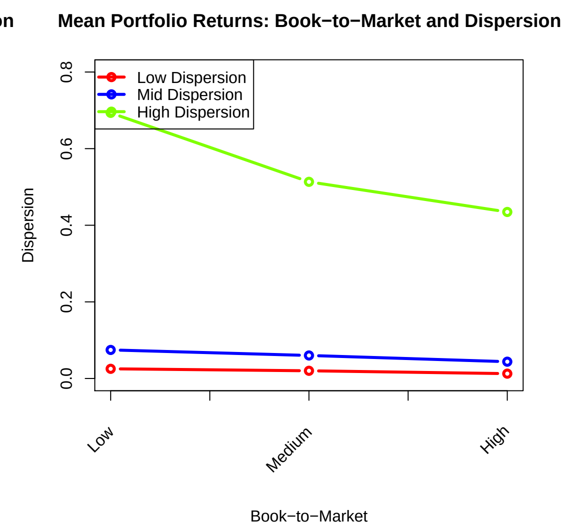
> the COMPUSTAT book value of stock holders' equity, plus balance sheet deferred taxes and investment tax credit (if available), minus the book value of preferred stock. Depending on availability, we use redemption, liquidation, or par value (in that order) to estimate the book value of preferred stock.

We calculate the book equity of all stocks yearly (with available data) and form a stock's monthly book-to-market ratio by dividing its yearly book equity by its market capitalization in that month. From our results we see that again, the difference in returns between low- and high-dispersion stocks in 4 the 9 portfolios are quite large. There still is a positive difference for the remaining five portfolios, but the effect is much smaller for large cap companies. And although, the difference in return between low- and high-dispersion stocks is larger in stocks with high book-to-market ratios (value stocks), it is not a strong relationship. However, stocks with high book-to-market ratios have much higher average *dispersions* than stocks with low book-to-market ratio suggesting that there is a positive relationship between book-to-market ratio and dispersion. This makes sense intuitively because the same level of disagreement in earnings per share should give a greater dispersion regarding the valuation of growth stocks than the valuation of value stocks because the differences in opinion on the book value of growth stocks are typically greater than that of value stocks. The results suggest that low-dispersion growth stocks (low book-to-market ratio) earn lower returns than low-dispersion value stocks (high book-to-market ratio) much like Karl B. Diether et al's (2002) main conclusion when they performed this portfolio test. The trends moving down from dispersion are generally the same but there is a noticeable difference between the trends for medium BM and high BM returns as you move across market size divisions. This is not too surprising given the discrepancy of the trends across size found in the previous portfolio test (the performance of the 2nd quintile of the market cap based portfolio deviated significantly). Here are four graphs, the top two showing original investor behavior, and the bottom two showing rational investor behavior:





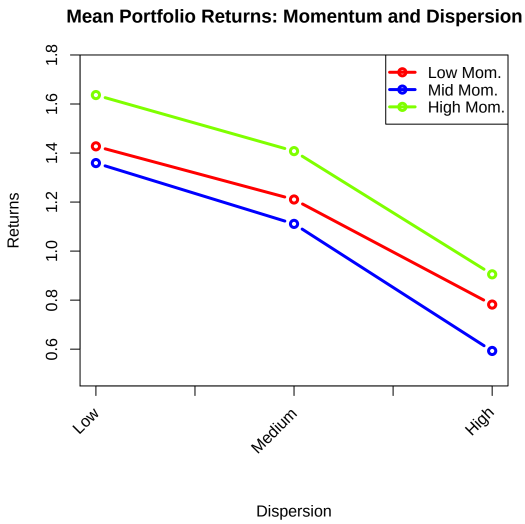


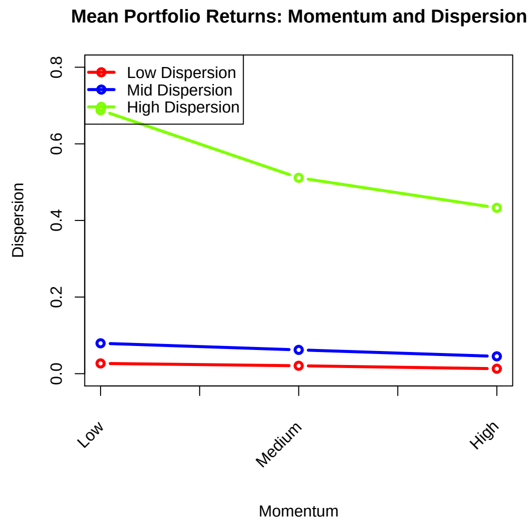


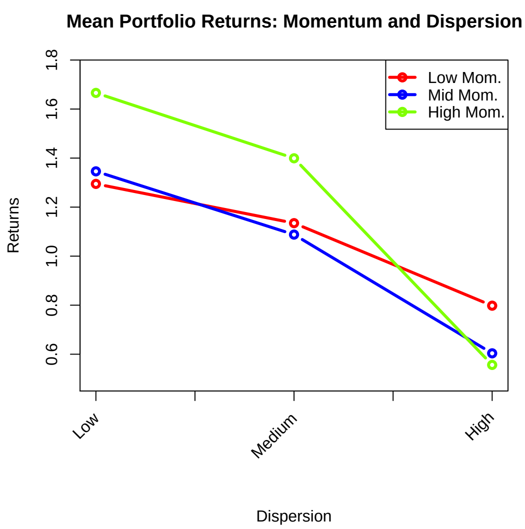
Interestingly, we see that as returns go down, stocks with high book-to-market values suffer greater losses in return with increases in dispersion. This suggests that taking the rational investor approach tampers with the dispersion effect such that it is not as uniform.

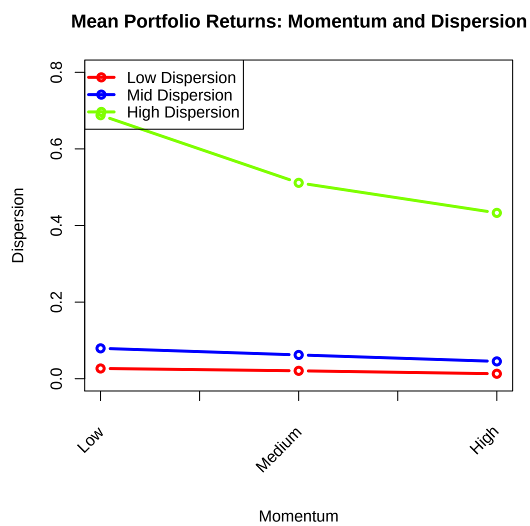
Size, Momentum, and Dispersion

Next, we investigate whether or not our results can be explained by a stock's previous performance. A poorly performing stock will most likely continue to perform poorly as documented by Jegadeesh and Titman (1993). We define momentum to be the past returns of a stock from months $t-12$ to $t-2$, as per Fama and French (1996). We create 27 portfolios again, sorting first by market capitalization of the previous month, momentum, and dispersion in analysts' forecasts from the previous month. There is a large difference in returns between low- and high-dispersion stocks, especially in small and poorly performing stocks, suggesting that the dispersion effect cannot by explained away by the previous performance of a stock if it was doing poorly. For this portfolio the trends as you move down dispersion levels are still apparent and the general trend of increased returns for winners are still visible. However the large spread that Karl B. Diether et al. (2002) finds between low and high dispersion losing stocks is not apparent. Here are four graphs, the top two showing original investor behavior, and the bottom two showing rational investor behavior:





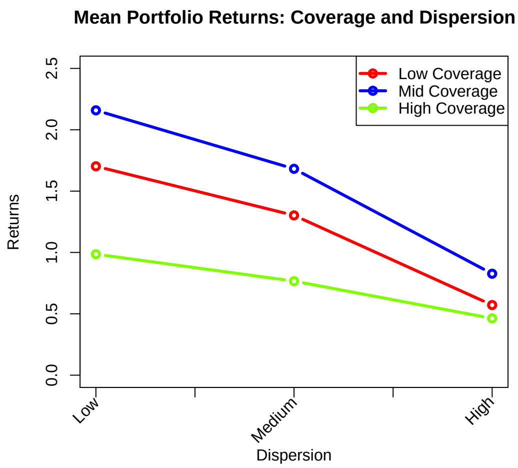


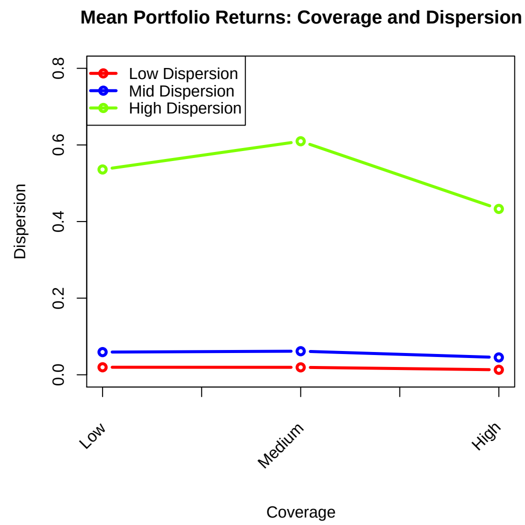


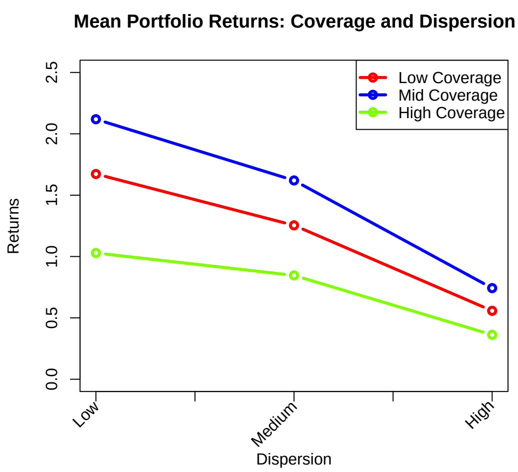
Again, we see that stocks predicted to do best in high dispersion by Diether et al. (2002), actually perform poorly when considering rational investors.

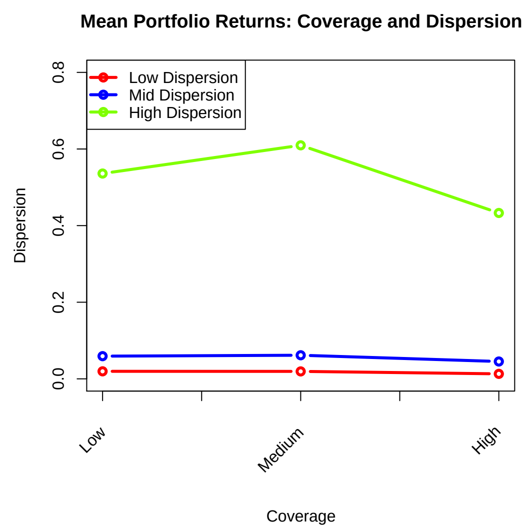
Size, Coverage, and Dispersion

In an extension of Karl B. Diether et al's (2002) work, we examine whether the extent of an analyst's coverage also affects the impact of dispersion on returns. Given the high correlation between analyst coverage and the size, or market capitalization, of a stock though we first sort by size to ensure we aren't simply capturing a size effect on dispersion again. Here are four graphs, the top two showing original investor behavior, and the bottom two showing rational investor behavior:





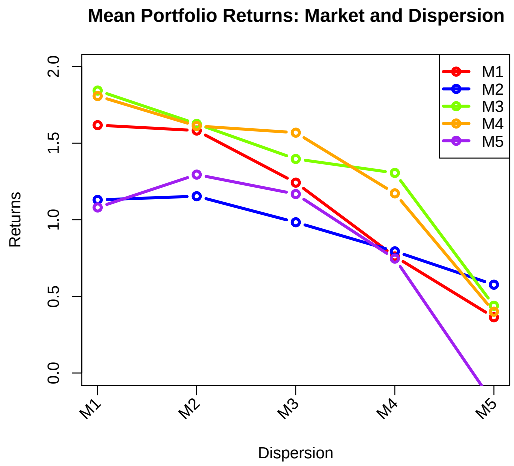


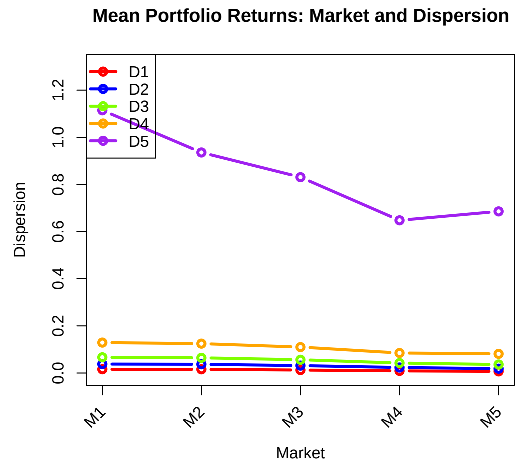


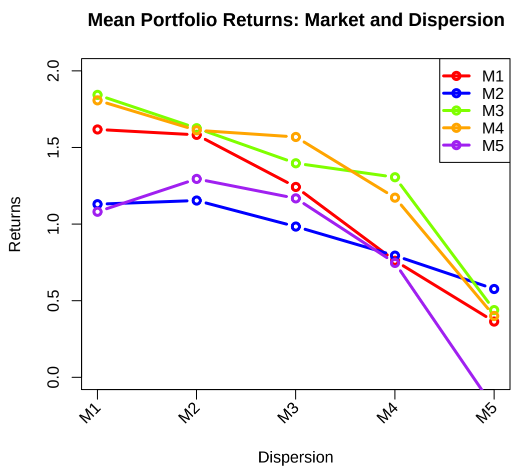
Here our results are surprising - stocks with a medium amount of coverage have better returns than those with a lot or a little coverage. Stocks with little coverage tend to be smaller and more unstable, so this makes sense. However, perhaps stocks with large amounts of coverage have greater variance, with a greater number of negative valuations than a stock with a medium amount of coverage; this is precisely the proxy for pessimistic investor opinion that lowers future returns.

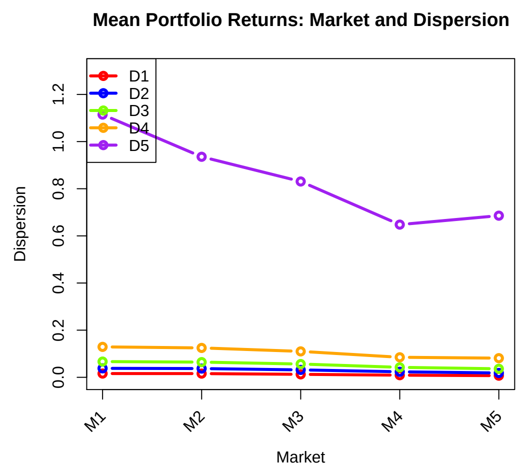
Market Performance and Dispersion

Given that investors have different behavior patterns with respect to how the overall market is performing we were interested in examining a new portfolio sorting test beyond Karl B. Diether et al. (2002) that focused on the relationship between market performance and the dispersion effect on returns. Here are four graphs, the top two showing original investor behavior, and the bottom two showing rational investor behavior:









The graphs here are very similar, and we see again, a negative relationship between dispersion and returns in all quintiles of overall market performance with best performance occurring when the overall market is doing poorly. Interestingly stocks in the highest dispersion quintile have a dramatically higher average dispersion than stocks in other quintiles. This is to be expected because the highest quintile has no upper bound, and likely to contain outliers. Because dispersion is a positive variable, there are no similar phenomena in the first dispersion quintile, that is, a tendency towards large negative values.

Conclusions

Our replication and subsequent extension of Karl B. Diether et al. (2002) led to several notable new findings on the predictive power of dispersion. Our multifactor time-series regression tests on dispersion based portfolios led us to conclude as Karl B. Diether et al. (2002) did that the Fama-French multifactor models do not have enough explanatory power to explain away the dispersion effect. However, there were deviations between Karl B. Diether et al's (2002) regression coefficients, GRS, and t-statistics and ours, suggesting that there were differences in our methods of data collection. Unfortunately, as of this writing, we have not been able to learn and implement the authors' exact data collection and parsing methods, but theoretically, a tuly robust relationship between dispersion and stock returns should have given us more similar trends, even with the inclusion of other factors. The significant deviations in the highest dispersion quintiles between the rational investor model and the original model suggest that wihle dispersion can help explain a portion of future stock returns, because realistic inclusions into our model significantly alter the systematic and ordered trends, the dispersion effect can be explained away. Similarly, while the regressive model described in the paper has strong t-statistics, it fails to account for many realistic components in market trading, even beyond a slighty more rational investor, including transaction costs and market unpredictability.

This said, in replicating the original portfolio tests performed in Karl B. Diether et al. (2002), we discovered that the negative relationship between dispersion and returns was readily apparent in all the portfolio tests and some of the main results that Diether et al. (2002) report such as low-dispersion growth stocks (low book-to-market ratio) earning lower returns than low-dispersion value stocks (high book-to-market ratio) and low-dispersion, low market capitalization stocks outperforming other stocks are all generally true in our tests as well. In our extensions, we also found that the performance of our portfolios was negatively correlated with overall market performance and, interestingly, stocks with a medium amount of analyst coverage out performed stocks with high coverage by a large margin. However, when we examined the effects of rational investment, we found that although our results were similar to the original results, in the cases of high dispersion, our results often differed greatly from those in Diether et al. (2002). We attribute this to the failure in dispersion to explain returns in more realistic situations because after extending our regressions to consider rational investor behavior, we found even stronger evidence to reject the multifactor model as an explanation for the dispersion effect. Looking back at the original three hypotheses, we reject the hypothesis that dispersion has predictive power because of strong GRS statistics against this, and we reject the the hypothesis that dispersion has a positive correlation with future stock returns because in all of our portfolio tests and regressions, there was a strong negative correlation. However, we accept the hypothesis that dispersion and future stock returns are negatively correlated cautiously because with the introduction of the rational investor, we found a smaller intercept with *stronger* GRS statistics. The intercept value, or α value represents both the error and unexplained effects in the Fama-French model such as dispersion, and when rational investor behavior is considered we have weaker t-statistics but smaller intercepts. This suggests that the stochastic error is comprising a greater portion of the α value and that some of the effect originally explained by the Fama-French factors or dispersion under original investor behavior is now being explained away by stochastic error. However, despite greater stochastic error, the stronger GRS statistic implies that there is greater evidence to reject the hypothesis that the Fama-French factors properly explain stock returns, so combined with the smaller intercept and the weaker t-statistics, we see that the dispersion effect explains much less of the excess returns than originally thought. This explains why portfolio results using rational investors and normal investors are essentially the same, with a few exceptions in high dispersion cases. When dispersion is high, the stochastic error is greatest and contributes most in explaining future stock returns. Thus because of lack of robustness in the portfolio results and decreased explanatory ability in rational investment strategy, we conclude that in actual trading situations, the predictive power of dispersion in analysts' earnings forecasts should be taken with caution.

Tables

For the Fama and French table pages, we provide two tables. The first is created using the original methods from the paper. The second examines mean analyst predictions. If the mean is less than zero, we assume that the rational investor would short that stock, so we switch the sign of the returns to indicate that if a stock's returns were negative, the investor would gain, and if the stock's returns were positive, the investor would lose. The second results are then calculated using these new returns. The remaining tables are replicated results using the methods in Diether et al. (2002).

Table I - Fama-French Three-Factor Model on Dispersion Equal-Weighted Quintiles

This table gives the coefficient estimates on the Fama-French three-factor model, $E(R_{it} - R_{Ft}) = b_{iM}(R_M - R_F) + s_i SML_t + h_i HML_t$, for the expected monthly return less the risk-free rate over five dispersion quintile portfolios. The market premium, MKT, is calculated using the CRSP NYSE/AMEX/Nasdaq value-weighted index. The size premium, SMB, is calculated using the market capitalizations of all stocks in the CRSP data base, and the value premium, HML, is calculated in the same way described above. The portfolios are formed by equal-weight averaging the returns of all stocks in a dispersion level in a given month. Stocks with a price less than five dollars are excluded from the data set, and t-statistics are Newey-West adjusted and are in parentheses. We also give R^2 values and the Gibbons, Ross, and Shanken (GRS) test.

		Factor Sensitivities			
Portfolio	α (%)	MKT	SMB	HML	Adj. R^2 (%)
D1 (low)	0.16	1.07	0.28	0.25	
	(1.12)	(34.64)	(2.49)	(2.36)	88.43
D2	0.18	1.09	0.34	0.20	
	(1.87)	(47.28)	(3.58)	(2.26)	92.53
D3	−0.11	1.11	0.50	0.18	
	(−1.35)	(42.81)	(6.09)	(2.98)	95.34
D4	−0.27	1.13	0.66	0.15	
	(−3.63)	(45.16)	(9.33)	(3.23)	95.87
D5 (high)	−0.91	1.15	0.85	0.18	
	(−7.36)	(44.08)	(14.03)	(3.95)	93.24
				GRS =	5.87

		Factor Sensitivities			
Portfolio	α (%)	MKT	SMB	HML	Adj. R^2 (%)
D1	0.14	1.05	0.26	0.31	
	(0.87)	(32.57)	(2.15)	(2.40)	83.84
D2	0.11	1.08	0.30	0.29	
	(0.86)	(41.91)	(2.88)	(2.51)	87.00
D3	−0.08	1.07	0.39	0.36	
	(−0.65)	(36.32)	(3.74)	(3.08)	85.66
D4	−0.40	1.01	0.40	0.44	
	(−2.94)	(31.31)	(3.55)	(3.69)	80.31
D5	−0.70	0.59	0.26	0.39	
	(−5.90)	(14.43)	(3.02)	(5.36)	65.29
				GRS =	7.59

Table II - Fama-French Four-Factor Model on Dispersion Equal-Weighted Quintiles

This table gives the coefficient estimates on the Fama-French four-factor model, $E(R_{it} - R_{Ft}) = b_{iM}(R_M - R_F) + s_i SML_t + h_i HML_t + m_i UMD_t$, for the expected monthly return less the risk-free rate over five dispersion quintile portfolios. The market premium, MKT, is calculated using the CRSP NYSE/AMEX/Nasdaq value-weighted index. The size premium, SMB, is calculated using the market capitalizations of all stocks in the CRSP data base, and the value premium, HML, is calculated in the same way described above. The momentum premium, UMD, is calculated by taking the difference between the returns of those stocks who had high returns in the months $t-12$ to $t-2$ and those stocks who had low returns in the same period. The portfolios are formed by equal-weight averaging the returns of all stocks in a dispersion level in a given month. Stocks with a price less than five dollars are excluded from the data set, and t-statistics are Newey-West adjusted and are in parentheses. We also give R^2 values and the Gibbons, Ross, and Shanken (GRS) test.

		Factor Sensitivities				
Portfolio	α (%)	MKT	SMB	HML	UMD	Adj. R^2 (%)
D1 (low)	0.32	1.07	0.30	0.19	−0.14	
	(2.40)	(37.43)	(3.36)	(1.88)	(−2.02)	89.43
D2	0.34	1.09	0.36	0.14	−0.15	
	(3.48)	(58.40)	(5.38)	(1.73)	(−2.54)	93.25
D3	0.07	1.10	0.52	0.12	−0.16	
	(0.99)	(51.26)	(8.93)	(2.21)	(−4.19)	96.42
D4	−0.10	1.13	0.68	0.08	−0.16	
	(−1.55)	(53.48)	(13.20)	(1.74)	(−4.99)	96.93
D5 (high)	−0.68	1.15	0.87	0.09	−0.21	
	(−5.46)	(50.14)	(22.17)	(2.10)	(−5.60)	94.23
					GRS=	2.83

		Factor Sensitivities				
Portfolio	α (%)	MKT	SMB	HML	UMD	Adj. R^2 (%)
D1	0.32	1.05	0.28	0.23	−0.17	
	(2.09)	(33.87)	(2.98)	(1.93)	(−2.07)	85.19
D2	0.31	1.07	0.32	0.21	−0.19	
	(2.54)	(45.81)	(4.49)	(2.00)	(−2.79)	88.68
D3	0.17	1.07	0.42	0.26	−0.23	
	(1.33)	(34.92)	(5.57)	(2.11)	(−3.22)	88.22
D4	−0.09	1.01	0.43	0.32	−0.29	
	(−0.62)	(26.52)	(6.87)	(2.79)	(−4.17)	84.81
D5	−0.44	0.58	0.29	0.29	−0.24	
	(−4.18)	(12.50)	(5.09)	(3.72)	(−4.29)	73.09
					GRS=	5.42

Table III - Mean Portfolio Returns by Size and Dispersion in Analysts' Earnings Predictions

The portfolios presented in this table are created on market capitalization levels and dispersion in analysts' estimates. Each month, all stocks are sorted and then assigned into a quintile of market capitalization, and within each quintile, further sorted by dispersion and assigned into a dispersion quintile to form 25 portfolios. Dispersion is defined as the standard deviation of analysts' current estimates over the absolute value of the mean. If the mean is equal to zero, then that stock is considered to have the highest dispersion. The portfolio returns are equal weighted in percentage returns for one month. We use data from February 1983 to December 2000, and are average monthly portfolio returns. D1 to D5 represent low to high dispersion quintiles, where S1 to S5 represent small to large stocks.

Mean Returns						
	S1	S2	S3	S4	S5	Avg.
D1 (low)	1.97	1.03	1.94	1.67	1.17	1.56
D2	1.92	1.01	1.60	1.55	1.31	1.48
D3	1.41	0.81	1.54	1.40	1.03	1.24
D4	1.20	0.74	1.32	1.14	0.87	1.05
D5 (high)	0.34	0.64	0.50	0.37	0.47	0.47
D1-D5	1.63	0.39	1.44	1.30	0.70	1.09
Mean Dispersion						
	S1	S2	S3	S4	S5	Avg.
D1	3.34	0.61	2.82	3.09	1.00	2.17
D2	0.02	0.02	0.01	0.01	0.01	0.01
D3	0.04	0.04	0.03	0.02	0.02	0.03
D4	0.07	0.07	0.06	0.04	0.04	0.06
D5	0.14	0.13	0.11	0.09	0.08	0.11
D1-D5	1.09	0.95	0.83	0.64	0.68	0.84

Table IV - Mean Portfolio Returns by Size, Book-to-Market, and Dispersion

The portfolios presented in this table are created on market capitalization levels, book-to-market values and dispersion in analysts' estimates. Each month, all stocks are sorted and then assigned into one of three groups of market capitalization, and within each third, further sorted by book-to-market ratio and assigned into one of three more groups, creating 9 portfolios. The stocks are then divided based on dispersion and assigned into a dispersion third to form 27 portfolios total. We remove stocks with a price less than five dollars. Dispersion is defined as the standard deviation of analysts' current estimates over the absolute value of the mean. If the mean is equal to zero, then that stock is considered to have the highest dispersion. The portfolio returns are equal weighted in percentage returns for one month. We use data from February 1983 to December 2000, and are average monthly portfolio returns.

Mean Returns									
	Low Book-to-Market			Medium Book-to-Market			High Book-to-Market		
Dispersion	Small Cap	Mid Cap	Large Cap	Small Cap	Mid Cap	Large Cap	Small Cap	Mid Cap	Large Cap
Low	1.84	2.23	0.95	0.89	2.69	0.89	2.40	1.73	0.84
Medium	1.26	1.62	0.71	0.46	2.41	0.62	2.11	1.24	0.94
High	0.52	0.60	0.53	−0.47	1.37	0.22	1.39	0.13	1.18
Low-High	1.33	1.63	0.42	1.36	1.32	0.68	1.00	1.60	−0.34
t-statistic	2.42	3.34	0.56	2.48	2.84	2.01	2.55	2.29	−0.46

Mean Dispersion									
	Low Book-to-Market			Medium Book-to-Market			High Book-to-Market		
Dispersion	Small Cap	Mid Cap	Large Cap	Small Cap	Mid Cap	Large Cap	Small Cap	Mid Cap	Large Cap
Low	0.02	0.03	0.03	0.02	0.02	0.02	0.01	0.01	0.01
Medium	0.07	0.08	0.07	0.06	0.07	0.05	0.05	0.04	0.04
High	0.68	0.77	0.64	0.51	0.61	0.43	0.42	0.41	0.47

Table V - Mean Portfolio Returns by Size, Momentum, and Dispersion

Each month, stocks are sorted into three portfolios based on market capitalization at the end of the previous month. Each of these portfolios is then divided into three momentum based portfolios, where momentum is calculated from the returns of stocks from month $t-12$ to $t-2$, where losers are the worst performing third and winners are the best performing third of stocks. Next, stocks are further sorted into three portfolios based on dispersion, giving 27 portfolios total. We remove stocks with a price less than five dollars. Dispersion is defined as the standard deviation of analysts' current estimates over the absolute value of the mean. If the mean is equal to zero, then that stock is considered to have the highest dispersion. The portfolio returns are equal weighted in percentage returns for one month. We use data from February 1983 to December 2000, and are average monthly portfolio returns.

Mean Returns									
	Losers			Neutral			Winners		
Dispersion	Small Cap	Mid Cap	Large Cap	Small Cap	Mid Cap	Large Cap	Small Cap	Mid Cap	Large Cap
Low	1.58	1.88	0.83	0.82	2.55	0.71	2.33	1.64	0.94
Medium	1.30	1.56	0.78	0.39	2.32	0.63	2.00	1.21	1.02
High	0.76	0.93	0.66	−0.33	1.64	0.47	1.50	0.27	0.95
Low-High	0.82	0.94	0.17	1.15	0.90	0.24	0.84	1.37	−0.01
t-statistic	1.53	1.98	0.23	2.08	1.92	0.74	2.10	1.96	−0.02

Mean Dispersion									
	Losers			Neutral			Winners		
Dispersion	Small Cap	Mid Cap	Large Cap	Small Cap	Mid Cap	Large Cap	Small Cap	Mid Cap	Large Cap
Low	0.02	0.03	0.03	0.02	0.02	0.02	0.01	0.01	0.01
Medium	0.07	0.09	0.07	0.07	0.07	0.05	0.05	0.05	0.04
High	0.68	0.76	0.63	0.52	0.60	0.42	0.42	0.41	0.46

Table V - Mean Portfolio Returns by Size, Number of Estimates, and Dispersion

Each month, stocks are sorted into three portfolios based on the number of estimates the stock received that month (coverage). Each of these portfolios is then divided into three market capitalization portfolios. Next, stocks are further sorted into three portfolios based on dispersion, giving 27 portfolios total. We remove stocks with a price less than five dollars. Dispersion is defined as the standard deviation of analysts' current estimates over the absolute value of the mean. If the mean is equal to zero, then that stock is considered to have the highest dispersion. The portfolio returns are equal weighted in percentage returns for one month. We use data from February 1983 to December 2000, and are average monthly portfolio returns.

Mean Returns									
	Small Cap			Mid Cap			Large Cap		
	Low N. Est.	Mid N. Est.	High N. Est.	Low N. Est.	Mid N. Est.	High N. Est.	Low N. Est.	Mid N. Est.	High N. Est.
Low	1.32	2.48	0.53	1.36	2.22	0.82	2.43	1.78	0.78
Medium	0.91	1.97	0.21	0.76	1.87	0.61	2.24	1.20	0.94
High	0.34	1.33	0.35	−0.06	1.13	0.31	1.43	0.02	1.13
Low-High	0.98	1.15	0.18	1.41	1.09	0.51	1.00	1.76	−0.35
t-statistic	1.86	2.29	0.25	2.40	2.53	1.60	2.61	2.54	−0.52

Mean Dispersion									
	Small Cap			Mid Cap			Large Cap		
	Low N. Est.	Mid N. Est.	High N. Est.	Low N. Est.	Mid N. Est.	High N. Est.	Low N. Est.	Mid N. Est.	High N. Est.
Low	0.02	0.03	0.02	0.02	0.02	0.02	0.01	0.01	0.01
Medium	0.07	0.08	0.07	0.06	0.06	0.05	0.05	0.04	0.04
High	0.67	0.80	0.61	0.51	0.62	0.42	0.43	0.41	0.47

Table VI - Mean Portfolio Returns by Market Premium and Dispersion in Analysts' Earnings Predictions

The portfolios presented in this table are created on market premiums and dispersion in analysts' estimates. All months are sorted into quintiles by market premium, and within each quintile, further sorted by that months average dispersion and assigned into a dispersion quintile to form 25 portfolios. Dispersion is defined as the standard deviation of analysts' current estimates over the absolute value of the mean. If the mean is equal to zero, then that stock is considered to have the highest dispersion. The portfolio returns are equal weighted in percentage returns for one month. We use data from February 1983 to December 2000, and are average monthly portfolio returns. D1 to D5 represent low to high dispersion quintiles, where M1 to M5 represent worst to best overall market performance. We also include a second table with fewer portfolios.

	Mean Returns				
	M1	M2	M3	M4	M5
D1 (low)	1.63	1.14	1.87	1.81	1.13
D2	1.62	1.18	1.65	1.61	1.28
D3	1.20	1.00	1.47	1.54	1.07
D4	0.74	0.68	1.45	1.30	0.83
D5 (high)	0.13	0.54	0.53	0.39	0.36
D1-D5	1.50	0.60	1.33	1.42	0.77
t-statistic	3.04	0.96	2.74	3.47	1.15
	Mean Dispersion				
	M1	M2	M3	M4	M5
D1	0.02	0.02	0.01	0.01	0.01
D2	0.04	0.04	0.03	0.02	0.02
D3	0.07	0.06	0.06	0.04	0.04
D4	0.13	0.12	0.11	0.09	0.08
D5	1.11	0.94	0.83	0.65	0.69

	Mean Returns		
	M1	M2	M3
Low	1.46	1.45	1.65
Medium	1.12	1.16	1.51
High	0.56	0.40	0.82
Low-High	0.90	1.05	0.82
t-statistic	1.55	2.44	1.46
	Mean Dispersion		
	M1	M2	M3
Low	0.02	0.02	0.01
Medium	0.07	0.06	0.04
High	0.70	0.52	0.44

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Appendix

Background on Risk and Return

When making an investment, an investor typically expects higher returns for riskier investments. Intuitively this follows since in order to convince an investor to accept greater risk, an investment must offer a higher *expected* return. This has held true historically, as the returns on treasury bills, a standard to measure the risk-free rate of investment, or the rate of return on a riskless asset, is much smaller than the returns on small stocks. The models we use were developed under the assumption that investors are risk-averse, and we employ this to make the relationship between risk and return more precise.

Proxies for Risk There are many measures of the riskiness of a stock. Volatility or the degree of fluctuation in a stock's returns has often been used as a measure of that stock's riskiness. The greater the historical standard deviation of a stock's returns, the greater the spread of future returns, so statistically an investor is much less sure of achieving the expected return or higher. However, the volatility of a stock itself is not the most important when measuring the riskiness of a stock.

Reduction of Systematic Risk If an investor could, for example, hold two different stocks, both with the same expected return, then it is less likely for *both* stocks to undergo extreme fluctuations at the same time, reducing overall volatility. This implies that the volatility of a single stock can be mitigated through diversification and that the remaining risk lies in the volatility of the stock relative to the other stocks in a portfolio. That is, as long as the stocks do not move with the same fluctuations as other stocks in the portfolio, overall volatility is reduced, without lowering expected returns, thus reducing riskiness overall. This kind of volatility is called unsystematic risk because it is a risk that affects a small number of stocks and can be diversified away. That is, this volatility does not co vary with the market as a whole, but rather is stock specific and can be reduced through diversification. However, with enough stocks in a portfolio, that portfolio tracks the overall market more and more until the portfolio's volatility matches the volatility of the overall market. Thus the investor is subject to the volatility of the overall market, and can only be compensated for this risk. This is called systematic risk and attempts to quantify the likelihood of a general stock market crash or some other financial disaster that affects an entire market. In other words, no amount of diversification can prevent systematic risk if the entire market is extremely volatile.

Market β as a Measure of Systematic Risk A stock's unsystematic risk is often measured using the historical standard deviation of its returns. However, the portion of a stock's return volatility that is systematic is measured by the degree to which the stock's returns vary relative to the returns of the overall market. A stock's systematic risk is the amount of risk that can be explained by overall market fluctuations, and is calculated as:

$$\beta_A = \frac{\mathrm{cov}(r_A, r_M)}{\sigma_M^2}$$

where β_A is an asset's market β, r_a is the return of the asset, r_M is the return of the market, σ_M^2 is the variance of the return of the market, and $\mathrm{cov}(r_A, r_M)$ is the covariance between the returns of the asset and the market. Typically, market betas are calculated using historical data, while using a large index such as the S&P 500 as the returns for the overall market. To determine the market beta of a portfolio, we weight each asset's beta by its market capitalization and average them.

Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model tries to systematically approach the relationship between the systematic risk of an asset and the expected returns of the asset. This model makes many assumptions, but the two largest and most apparent are the assumptions of investor behavior and that there is only a single risk factor. The model assumes that investors only care about expected returns and volatility, and as rational consumers, attempt to maximize expected return for a given level of expected volatility. All investors are assumed to hold identical beliefs regarding the amount of reward for a given amount of risk in the market. The model also assumes that there is only one risk factor, systematic market risk that cannot be diversified away. Thus the corresponding expected return of a security is calculated solely using a security's market beta.

Let us take several corner case scenarios to investigate the formulation of the Capital Asset Pricing Model. First, let us examine an asset that has no volatility. The asset has no risk, and its returns do not vary with the market, so its beta is equal to zero and its expected return is equal to the risk-free rate of return:

$$E(r_A) = r_F$$

where r_A is the return of an asset and r_F is the risk-free rate of return.

Next, imagine an asset that perfectly tracks the market's volatility. This asset has a beta equal to one and, by our supposition, has expected returns equal to the market:

$$E(r_A) = E(r_M)$$

If we then take an asset with greater volatility than the market, or with a beta greater than one, we expect greater expected returns from this asset than the market to compensate for increased risk. If we take the expected return of the market portfolio beyond the risk-free rate, $(E(r_M) - r_F)$, the equity risk premium, we form the CAPM equation:

$$\boxed{E(r_A) = r_F + \beta_A(E(r_M) - r_F)}$$

The original CAPM equation captures the effect of the relationship between market return, how closely an asset tracks the market volatility, and that asset's expected returns, where the asset earns the risk-free rate of return plus some amount corresponding to the relative risk of the asset to the market overall. If we rearrange the CAPM, we see that:

$$\beta_A = \frac{(E(r_A) - r_F)}{(E(r_M) - r_F)}$$

so it is clear that beta is the ratio of the expected returns of an asset less the risk-free rate to the expected returns of the market again, less the risk-free rate. Again, these equations are created under the assumption that the β value and market returns less the risk-free rate perfectly explain an asset's returns. More accurately, we should write the CAPM with an error term that varies with time:

$$E(r_A - r_F) = \beta_A(E(r_M) - r_F) + \epsilon_t$$

where the error term is distributed as a Normalized Gaussian. Thus the stochastic component of the CAPM is given by:

$$\epsilon_t \sim \text{Normal}(\mu_t, \sigma_t)$$

and the systematic component is given by

$$\mu_t = \beta_A(E(r_M) - r_F)$$

and σ_t is simply the standard deviation in overall market returns in the time period t.

Application Given that the CAPM predicts the expected return of an asset or portfolio, it becomes a useful tool to analyze the effect a certain characteristic on the portfolio. If, for example, a portfolio is created by a fund manager, then the manager could point to the difference between the predicted return from the CAPM model and the actual return as his or her added value to the portfolio. In financial literature, this excess return, or intercept is called the α value. Much in the same way we could evaluate whether or not a low dispersion portfolio leads to excess returns, and this is essentially what we do in our regressions in the coming sections.

In order to evaluate α, we use the CAPM model and run a regression on three time series of data. A time series simply refers to a sequence of data measuring one statistic or attribute as it changes over a given frequency of time. We need returns for the stocks in the portfolio we are investigating, the overall market returns for the same period, and the risk-free rate of returns for the same time period as well. The equation we examine is given by:

$$r_A = r_F + \beta_A(r_M - r_F) + \alpha$$

where of course α is the added value of the attribute of the stocks that we are investigating. If we rearrange the formula, we can create a linear model for the market rate of return with the intercept being α:

$$E(r_A - r_F) = \alpha + \beta_A(r_M - r_F)$$

And β is the slope of the line.

Criticisms In empirical tests, the CAPM's predictions are often wrong with roughly 15% of the variation in observed returns unexplained. Also, researchers have found other predictive risk factors that significantly influence the expected returns of a stock. Because one of the CAPM's most basic assumptions is the aggregation of all risk into a single term, by simply stratifying risk into different categories beyond market risk, such as bankruptcy risk, size and value, we can create a more predictive model. We can also see more clearly the weights that each factor holds on expected future returns.

Fama-French Multifactor Models

In 1992, Eugene Fama and Kenneth French published their findings on possible CAPM extensions. They found that the value of a company (its book-to-market ratio) and the size of a company (its market capitalization) are two of the most significant factors in explaining the returns of stocks. SMB stands for Small Minus Big and is called the size premium. It was implemented to include the extra returns historically paid out to stocks with smaller market capitalizations. In Fama and French (1992), a month's size premium is calculated as the average return of the smallest 30% of stocks minus the average return of the largest 30% stocks in a month. If SMB is positive, then small capitalization stocks outperformed large capitalization stocks in that month. HML stands for High Minus Low is the value premium that investors get when they invest in companies with high book-to-market ratio, basically the ratio between the valuation of the company made by accountants and the valuation of the company the public makes. The HML is calculated by taking the returns of the stocks among the top 50% book-to-market ratio in a month minus the returns of the stocks among the bottom 50%. A positive HML means that value stocks, or stocks with high book-to-market ratios, have outperformed growth stocks in a particular month. SMB and HML are popular factors to use because in addition to market risk, these three variables explain about 95% of the variance in returns. SMB represents the risk that faces smaller companies - perhaps they are less able to absorb negative shocks than larger companies. Historically, smaller companies have had greater volatility in returns than larger companies. HML suggests that as public valuation of a stock goes down the book-to-market ratio goes up, public opinion regarding its future returns goes down. Since these companies might have experienced hardship that caused public opinion to go down, it seems reasonable that they would have greater risk of difficulty. Some hesitate to call HML a risk factor, because the scenario just outlined doesn't seem to cover all possibilities. Regardless, SMB and HML combine with market returns to more effectively explain the returns of a stock. Please refer to the *Regression Tests* section of the *Methodology* for a further description of the Fama-French three factor model and an introduction of the Fama-French four factor model.